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THE

PRISMOMETRIC SCALE.

WITH ELEVEN ORIGINAL DIAGRAMS.

By CHARLES F. PRENTICE,

OF NEW YORK.

Reprint from THE AMERICAN JOURNAL OF OPHTHALMOLOGY, October, 1891.

Dr. ....

Dear Sir:

The accompanying Prismometric Scale is presented to you in the hope that you will appreciate the advantage which it offers for securing a better understanding between yourself and your optician, respecting the accuracy of his execution of your prescriptions for prisms which are within the limit of application for spectacles. Use of the scale to attain accurate duplication within this limit will not necessarily involve rejection or disregard of the conventional degree system, with which, to this extent, it is approximately reciprocal. It is also the only scale bearing a direct relation to the decentration of dioptral lenses.

As the present edition of the scales is limited, the original lithographic stone will be preserved, in anticipation of the profession creating a further demand for another on the part of opticians.

Yours respectfully,

Charles F. Prentice

178 Broadway, NEW YORK.

December, 1891.

THE PRISMOMETRIC SCALE.

With Eleven Original Diagrams.

BY CHARLES F. PRENTICE, NEW YORK.

During the past two years "The Metric System of Numbering and Measuring Prisms"[1] has been a subject of considerable discussion, although the exact nature of its unit, the prism-dioptry, does not seem to have been generally understood, while its practical advantages to opticians, "of whom accurate work is expected," have been wholly disregarded in some recent criticisms, in which it has been compared with Dr. Jackson's and Dr. Dennet's equally as scientific though less *convenient* systems. It is, therefore, now proposed to call attention to a still more simple feature of the metric system, with further explanations, yet with the understanding that the reader is familiar with its general principle and applications as originally explained.

The prismometric scale, preferably drawn upon heavy paper or card board, consists of a line of gradations, "6 centimeters apart,"[2] which are indicated by heavy vertical lines, with an in-

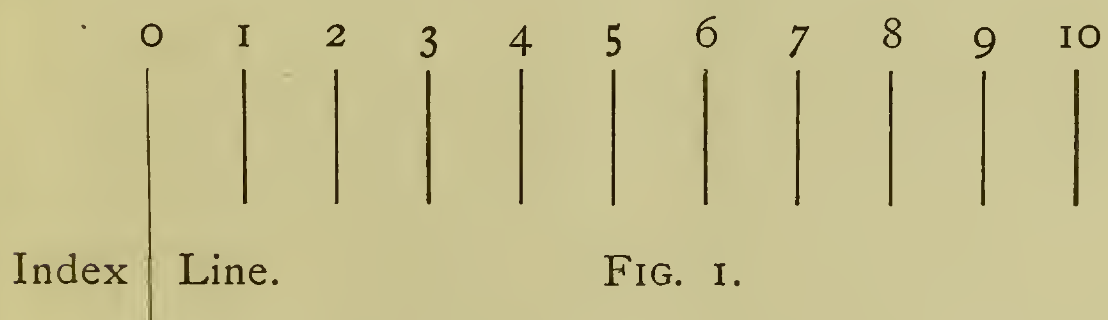

Fig. 1.

dex-line at zero, longer than the rest, as shown in Fig. 1, which

[1]A Metric System of Numbering and Measuring Prisms. By Chas. F. Prentice. Archives of Ophthalmology, Vol. xix, Nos. 1 and 2, 1890, and Vol. xx, No. 1, 1891.

[2]See Archives of Ophthalmology Vol. xix, No. 2, 1890.

being just six times greater than the "coarse centimeter scale" referred to in my first paper, is intended to be placed at a six times greater distance, or "6 meters" from the eye; when simple prisms may be measured by it according to the manner originally set forth.

The *average* deflections produced by our commercial prisms, marked 1° to 5°, will be found to correspond closely to this scale up to the fifth division.

In applying the scale to the measurement of sphero-prismatic lenses, it is evident that the index-line will be rendered more or less indistinct in viewing it through such a lens, so that the lenticular element of the sphero-prismatic lens will require to be fully neutralized by a *contra-generic* lens of the same power, when, by shifting the neutralizing lens from right to left, it will be possible to secure a position for it which will leave us the prismatic deflection which it is sought to attain by the inherent prism of the entire combination.

The procedure is best explained by the following example: The optician being requested to grind a sphero-prismatic lens of $+3$D. sph. $\bigcirc$ 2 prism-dioptries, selects from his stock a prism which is *rough* on one side, and which he consequently is obliged from its *marking*, to take for granted is a prism of 2°. He then grinds the rough side to $+3$D. spherical, when, according to the old method, he naturally assumes that he has accomplished the full object of his purpose. It is now suggested that he carefully determine the optical center of a *concave* lens of 3 dioptries, and mark this point with an ink dot, placing the opposite side of this neutralizing lens in contact with the spherical side of the sphero-prismatic lens which it is desired to measure. He is next requested to hold the entire combination before his eye, at exactly 6 metres from the scale, the precaution being taken to have the base-apex line of the sphero-prismatic lens horizontal, with the base to the left, and in such a manner that the upper edge of the entire combination covers only the lower half of the pupil. The index-line observed through the lenses will then appear to be displaced toward the right, relatively to the graduations as

seen through the uncovered upper portion of the pupil. In the event of the index-line appearing to be displaced more or less than the required graduation marked "2," the operator has only to shift the neutralizing lens carefully to the left or right, until the index line exactly cuts the second graduation. Care should be exercised not to change the position of the sphero-prismatic lens during this act, and while in this position, an ink dot should be placed on the sphero-prismatic lens, precisely opposite to the dot on the neutralizing lens. The former then indicates the point which should form the center of the glass in the spectacle frame.

The reasons for this will be obvious from a consideration of the following figures:

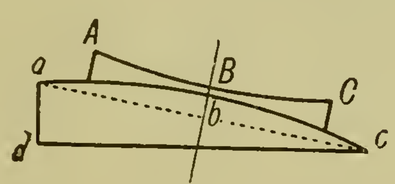

FIG. 2.

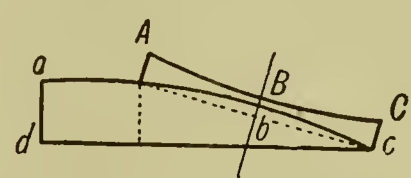

FIG. 3.

The concave lens A B C in Fig. 2, with its center at B, neutralizes the convex lens a b c, thus securing the *effect* of a prism a c d, *just at the opposite points* B b. By shifting the neutralizing lens, as shown in Fig. 3, the effect of a prism of greater angle is obtained. It is, consequently, possible, within reasonable limits, by this means to correct any inaccuracy which may have existed in the original *rough* prism. The same effect is obtained in sphero-cylindro-prismatic lenses, by neutralizing the cylindrical element with an additional and carefully adjusted *contra-generic* cylindrical lens, though this is naturally a little more difficult.

I shall preface a further discussion of this question with a few simple optical definitions, which I hold to be indispensable to a thorough understanding of the subject, and which, much to my regret, and for reasons too obvious to mention, were not presented by me in my previous papers in the *Archives of Ophthalmology*.

1. The optical center of a lens is a point situated upon a line called the *optical axis, which must be perpendicular to both the anterior and posterior surfaces of the lens.*

2. Direct Pencils.—Rays of light which are emitted from a luminous point upon the optical axis will be refracted and directed to a conjugate point upon the same axis, it being specifically noted that the axes of the incident and refracted pencils of light and the optical axis of the lens *must* coincide.

3. Oblique Pencils.—In any case where the axis of the incident cone of light does not coincide with the normals to the surfaces of the refracting medium, whether it be a lens, prism or plate, the refracted pencil will no longer be a circular cone of light; but it will be a pencil bounded by a surface penetrating the medium and defining its illuminated area, besides intersecting two focal lines, which are at right angles to each other and the axis of the refracted pencil.

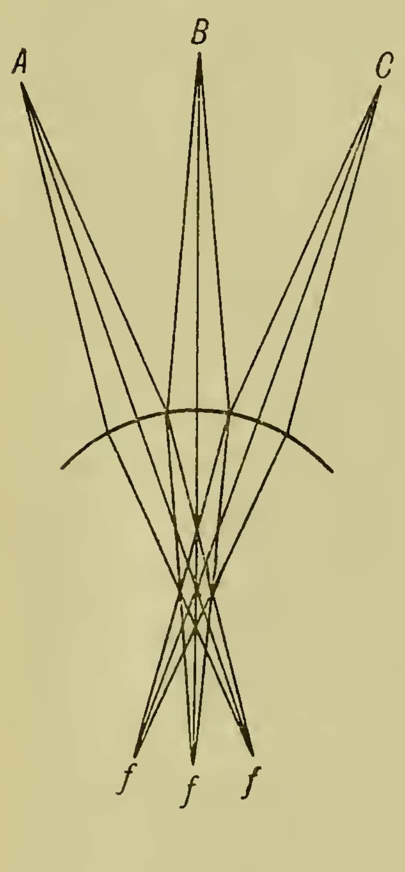

Fig. 4.

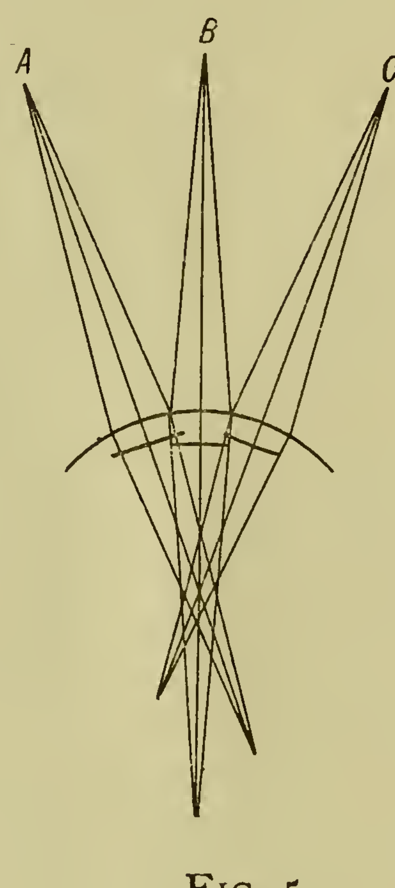

Fig. 5.

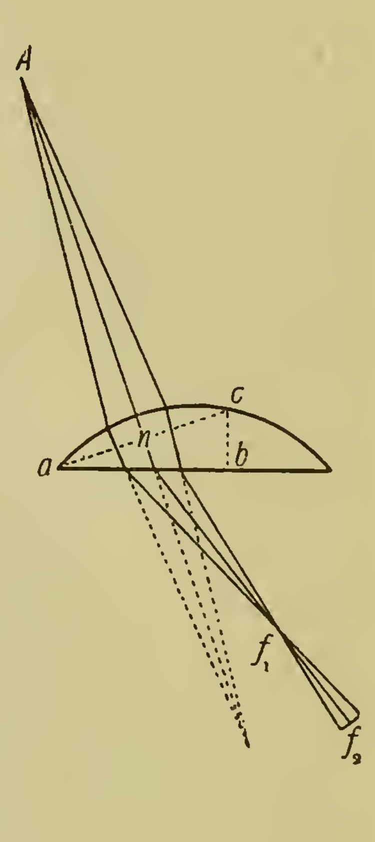

Fig. 6.

The same laws apply to the *reflection by spherical surfaces* of direct and oblique incident pencils of light, and their mathe-

matical elucidation is given by Profs. R. S. Heath and W. Steadman Aldis, in their recent exhaustive treatises on Geometrical Optics.

In illustration of the above definitions, let the curved line in fig. 4 represent the spherical surface of a medium with a greater density than air, when perpendicularly incident conical pencils of light, projected upon it from successive points A, B, C, will have their respective conjugate foci, f, upon the correlative radii with which the axes of the incident pencils coincide. If the refracted pencils, *within* the medium, are to have *focal points outside* of the medium, the axes of these pencils will have to be *perpendicularly* intercepted by the second surfaces as shown by the heavy lines in fig. 5; and in the event of the second surface occupying an oblique position, a b, fig. 6, with respect to the pencil A, the medium must be considered as a lens, having its optical centre upon the axis An of the incident pencil, with the prism a b c added to it.

The circular cone of light, *within* the medium, will then project an elliptical area of illumination, E, fig. 7, upon the second surface, as the axis of the pencil is here *oblique*, and the refracted pencil ceases to be a circular cone, projecting itself outside of the medium as an *astigmatic* pencil, of which f_1 and f_2 are the focal lines at right angles to the axis, the whole being deflected toward the base of the inherent prism P.

While this optical phenomenon, which in this case we may term a sphero-cylindro-prismatic action, may be new to many, it has been known to physical science since Kummer, in 1860, first called attention to the theory by which it was mathematically demonstrated. Its significance to ophthalmological science may, perhaps, be treated of in the future.

The fact, however, may be experimentally, though roughly, demonstrated by placing a plano-convex lens of 8 dioptries directly between a light at 10 or even 20 feet, and a screen receiving its image. On interposing a prism of 20°, for example, with its base down, and in a manner to insure contact of the plane faces of the glasses, the image will be observed to change both its form and position upon the screen. By draw-

ing the screen slightly nearer to the lens, a horizontal though imperfectly defined line corresponding to f_1 will become manifest, and by increasing the distance between lens and screen a vertically elongated looped figure, closely resembling a line at f_2, will appear.

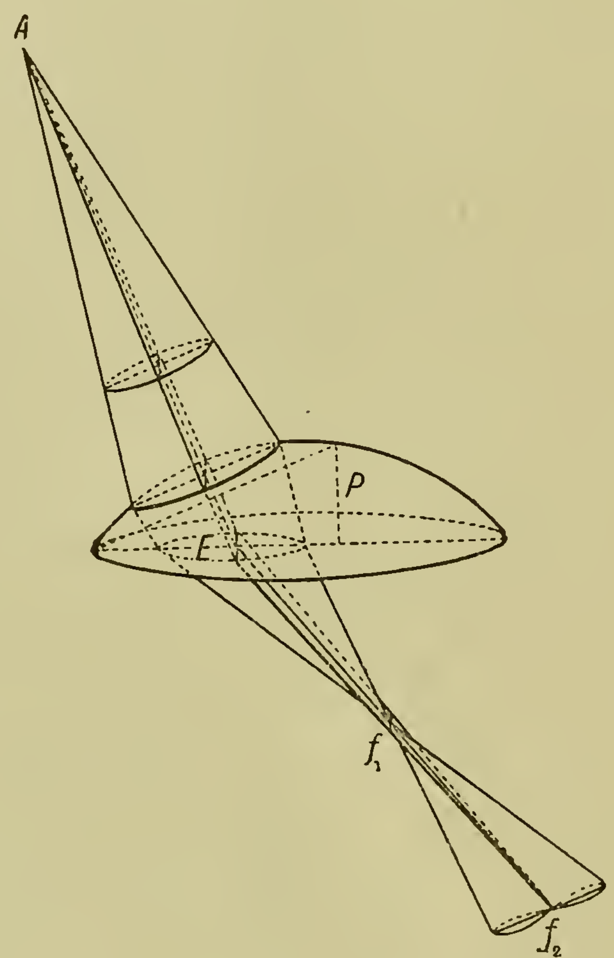

FIG. 7.

When a circular cone of light, C, fig. 8, from a short finite distance falls *obliquely* upon the face of a simple prism, we again have an elliptical area of illumination, and the refracted rays *within* the medium will assume a direction as if emitted from the focal lines v_1, v_2, reaching the second surface of the prism, and being refracted by it to the eye at E, as if projected from the lines V_1, V_2, on the opposite side of the prism.

There is one exception to this result, and that is when the axis of the incident pencil assumes a direction which is subject to minimum deviation, in which event the emergent pencil will appear to diverge from a *point*, at the same distance

from the anterior surface as the original source of light C. In the case of a plate, the emergent pencil will also be of astigmatic form, with the difference that it will appear to proceed from a pair of focal lines located upon an axis *parallel* to the axis of the incident pencil.

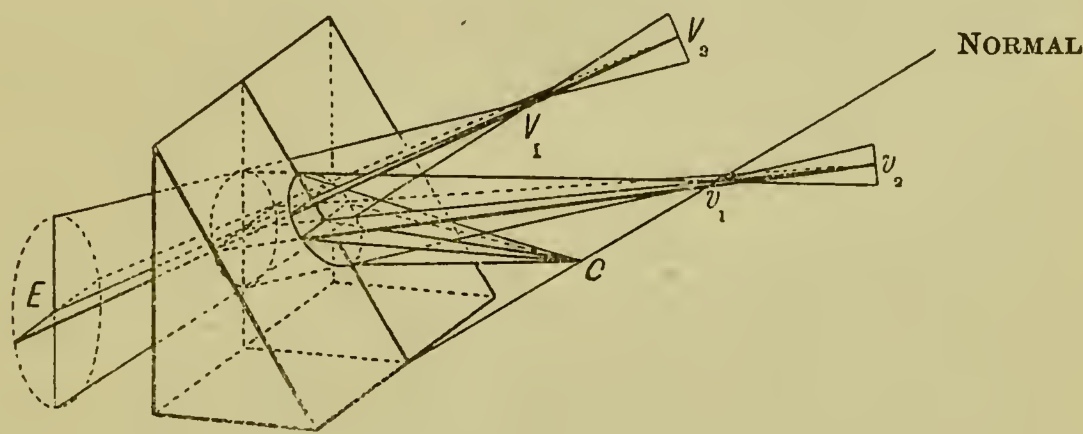

FIG. 8.

This sphero-cylindro-prismatic action, on the part of a simple prism may be experimentally demonstrated in the following manner. Having constructed the figure M O (to the left in fig. 9) in which the width of the principal lines is say 2 inches, and the distance apart of the perpendiculars is about 24 inches,

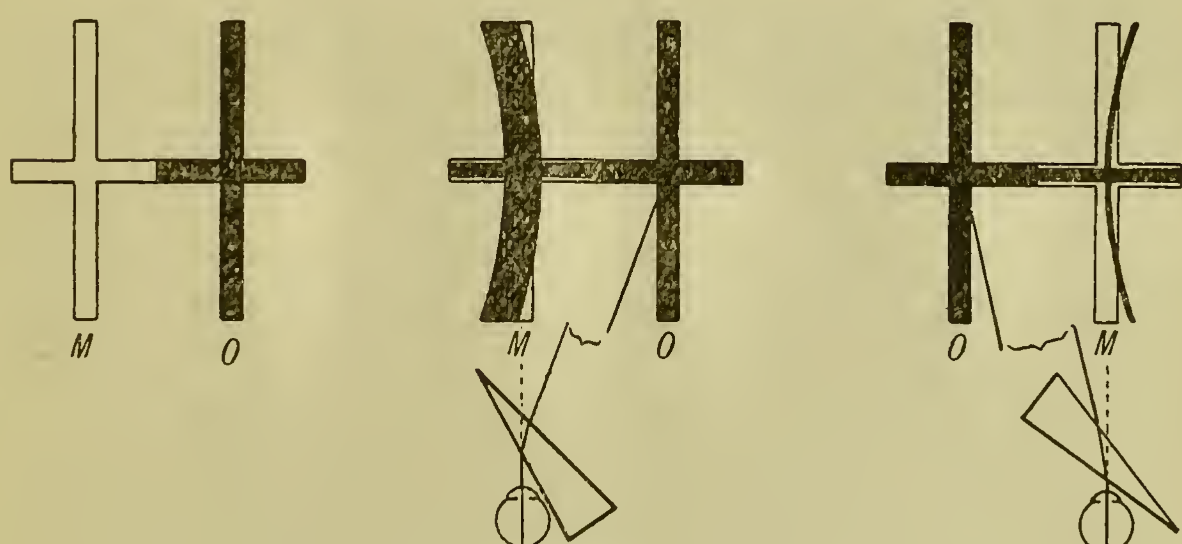

FIG. 9.

place it at right angles to the line of sight, at a distance of about 6 feet from the eye, before which a prism of 10° is given

considerable inclination to the visual axis, with its base in or out, and as shown in the diagrams, fig. 9, which should suffice to indicate both methods and their results. The eye in each instance is to be placed directly opposite to the figure M. With these facts in mind we may return to our subject of measurement.

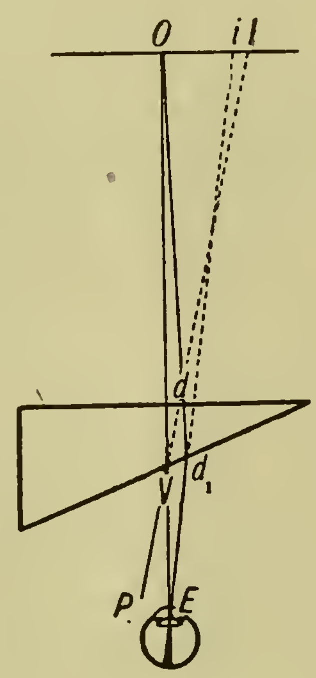

FIG. 10.

In fig. 10 the relative positions of the object of fixation O, the prism, and the eye are shown. It is evident that the perpendicularly incident axis O V of the conical pencil of rays emitted by the object O coincides with the visual axis, and that the axis of the refracted pencil V P *does not* enter the eye, although it *does* define the deflection O I which it is desired to ascertain. The axis of the refracted pencil, d_1 E, which *does* enter the eye, however, will result from that incident pencil whose axis is *oblique* relatively to the normal at d, and it will therefore be a ray approaching direction for minimum deviation and will consequently suffer less refraction than the refracted pencil whose axis is V P.

Now if, as is the case with the prismometer, the observer reads the deflection O i at the finite distance marked, say "10", upon the graduated bar, it is evident that an error will be com-

mitted, since 10 times O i will be less than 10 times O I[3]; yet this seeming weakness in my previous papers has escaped detection by the critics of the prism-dioptry system, and for the consolation of whom let it now be said that there could have been no reasoning so clever or ingenious on their part as to have made this error any the less apparent, *even in a prism of 10°*, by merely contrasting the differences between arcs, sines and tangents, in a choice for the unit of measurement; besides, a mere consideration of the well known relative goniometrical values of these has not hitherto been pertinent to the discussion, since the proposed unit, the prism-dioptry, is not a goniometrical unit, but an *optical unit.* The desire to multiply any *unit in optics* should be curbed by a knowledge of the fact that all the fundamental optical laws are based upon the assumption and acceptance of *values of limited magnitude*, and that there is therefore apt to be a point where *unreasonable* multiplication of an optical unit will contradict the actually existing optical phenomenon. *A warning to this effect was given in speaking of the decentration of lenses* (see page 129 of my second paper).

Even *thickness*, a dimension which we are taught to neglect with respect to ophthalmic lenses, becomes an appreciable factor in prisms above 10°, when we attempt to measure their deflection at short finite distance. This will be apparent from the following considerations.

It has been shown that the ray, which in the nearest limit reaches the eye, is the axis O d, fig. 11, of an *oblique* pencil, being refracted within the prism A B C from d to d_1, and thence in air to the eye E, which projects it to i, upon the scale O I. For a given thickness of prism, this is the *only* pencil which will be received by the eye, since, if we increase the thickness by allowing the plane $A_1 B_1$ to represent the anterior surface of the prism, the original incident axis O d will be refracted at v instead of d, when the axis of the refracted pencil will traverse

[3]This will be equally true for measurements taken from an arc at short finite distance.

the path vv_1 P_1 to the left of and parallel to dd_1 E. The refracted pencil which would enter the eye, for the indicated *increased* thickness, could only accrue from an *increased* obliquity of the incident axis Ou. The latter would therefore even more closely approach position for minimum deviation, from which we are to conclude that the deflection noted upon the scale OI by the eye will be *least near the base* and consequently greatest near the apex of the prism.

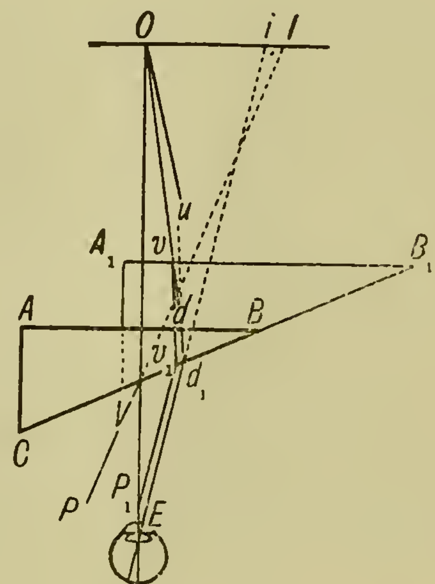

FIG. 11.

This is really proven to be the case by experiment with the prismometer. At the distance marked "10" upon the bar, the index-plate-reading near the base of a 22° prism, $1^1/_4$ inches square, is found to be 1.79, whereas at the feather-edged apex it is 1.89, so that the prism in the former instance measures 17.9 P D, while in the latter it is 18.9 P D. The same prism measured by the prismometric scale at 6 meters reads 20 P D. The error committed by measurement through the apex, at short finite distance, is therefore 1.1 P D, while the increased thickness at the base still further increases the error by 1 P D. The error will consequently be least, *in prisms of high degree*, when readings on the prismometer are taken at the apex of the prism, and it will be reduced to a minimum, *throughout the principal refracting plane*, when the deflection is measured for

pencils which are *perpendicularly incident to all points of the prism-surface,* that is to say, when the pencils of light are *cylindrical,* and which will practically be the case when the object of fixation, *a line,* is situated at 6 meters distance. In fact it will be better to measure all prisms above 10° at this distance.

This sharply defines both Dr. Burnett's and my own reason for advocating the tangent plane for the position of the scale, since it will be infinitely more *convenient* to place such a scale upon a flat wall, with which every office and workshop is provided, than to *contrive* an arc of 6 meters radius.

Other advantages of the scale at a 6 meter distance were mentioned in my second paper, when referring to hyperphoria.

The above facts do not lessen the value of the prismometer, which I have repeatedly and specifically represented as being of importance to opticians in filling oculists' prescriptions, in which the prisms do not exceed 5°, and by reason of which the error is so slight as to be inappreciable, yea, even in a prism as high as 8°, when an attempt is made to verify measurement by the prismometric scale at 6 meters.

It was also to be supposed that all oculists and opticians would not provide themselves with prismometers, in which event it was further anticipated that the prismometric scale would have to be resorted to, and more particularly now that hair-splitting fractions of the unit are not considered to be of value.

A more simple and convenient means of verifying the opticians' work could not be placed in the hands of the oculist.

The prism dioptry does not exclusively depend upon trigonometrical laws, nor rest solely upon the adoption of a specific instrument, but it is based upon a principle which is easily understood and capable of being practically applied within the confining limits set by the fundamental laws of optical science; and it must further be apparent that the generally irrelevant criticisms which have appeared in print have not, thus far, proven anything to the contrary; while it must be equally

clear that this paper contains a review of the optical laws and phenomena which must be considered in the choice of a unit, and that these will require to be thoroughly understood, before anyone can undertake a rational criticism of the subject. However, it is admitted that a perpetuance of the present degree system, together with the commonly accepted approximations which must accompany its application in practice, will obviate such pains being taken.

THE RELATION OF THE PRISM-DIOPTRY TO THE LENS-DIOPTRY OF REFRACTION.

In a previous paper* it has been demonstrated, theoretically, that

a lens, decentered one centimeter, will produce as many prism-dioptries as it possesses lenticular dioptries of refraction.

The following simple experiment will serve to prove it.

Take any pair of *contra-generic* lenses† of equal power, ranging between one and ten dioptries, and carefully mark their optical centers with ink-dots. Then place the lenses upon each other so that the ink-dots are exactly *one* centimeter apart, when the reading taken from the Prismometric Scale will be found to correspond with the dioptral power of the lenses. It is consequently evident that the scale could also serve to determine the refractive power of the lenses, in the event of their numerals of refraction being unknown.

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*A Metric System of Numbering and Measuring Prisms, By Charles F. Prentice, *Archives of Ophthalmology*, Vol. XIX, No. 1, 1890.

†One lens being convex, the other concave.